Is T_2 necessarily shorter than T_1?

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If you are familiar with MRI (or NMR in general), then probably also with the relaxation time constants T_1 and T_2. These tissue-specific (or substance-specific) constants describe how fast the nuclear magnetization returns to its equilibrium value, M_0, after excitation by a pulsed radio-frequency (rf) field. Shortly summarized, T_1 describes the exponential recovery of the longitudinal magnetization M_L (i. e., of the magnetization parallel to the external static magnetic field B_0), and T_2 describes the exponential decay of the transverse magnetization M_T (that is precessing in the plane orthogonal to B_0).

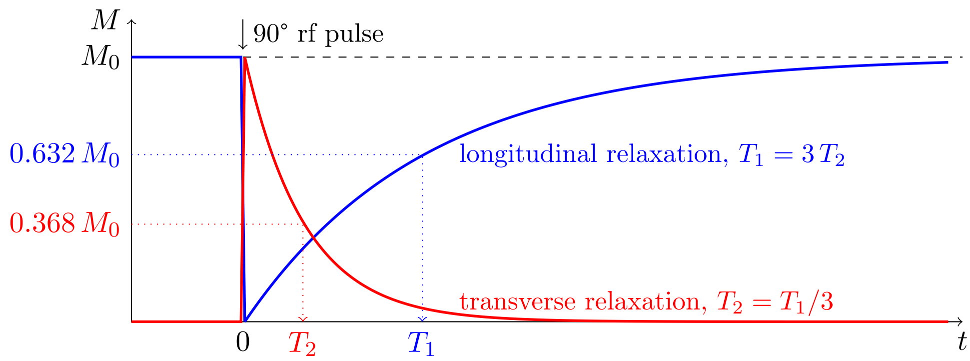

Figure 1: Longitudinal (blue) and transverse (red) relaxation of nuclear magnetization after a 90° rf pulse

Typically (as shown in the first figure), T_2 values of tissue are considerably lower than T_1 values, i. e., the transverse magnetization decays quicker than the longitudinal relaxation needs for recovery. For most tissues *in vivo*, T_1 varies between about 300 ms and 3 s, while T_2 varies between about 10 ms and 200 ms. Longer T_2 relaxation times (up to about 3 s as well) are found for liquids.

So one may ask if there are good physical reasons for T_2 values being shorter than or – at most – equal to T_1. As a physicist, I'd start with checking some extreme cases, e. g., assuming that T_2 is *much* longer than T_1, i. e. $T_2 \gg T_1$. Then the longitudinal magnetization can fully recover while at the same time some transverse magnetization

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would be preserved. As a result, the magnitude of the total magnetization $\sqrt{M_\mathrm{L}^2 + M_\mathrm{T}^2}$ would become greater than the equilibrium value M_0 – which is physically impossible.

To analyze these properties of T_1 and T_2 in more detail, longitudinal and transverse relaxation can also be plotted together in a diagram showing the transverse magnetization on the horizontal axis and the longitudinal magnetization on the vertical axis. These diagrams show the evolution of the magnetization (for experts: in a rotating frame of reference) after a 90° rf pulse (all trajectories start at the lower right corner of the diagram). The left-hand side of the following figure shows three cases $T_2 = T_1/2$ (blue), $T_2 = T_1$ (green), and $T_2 = 2\,T_1$ [sic!] (cyan); and all three curves show a "benign" behavior in that they lie in the shaded area below the black circle segment $M_\mathrm{L}^2 + M_\mathrm{T}^2 = M_0^2$. This means that the magnitude of the total magnetization vector is always smaller than M_0.

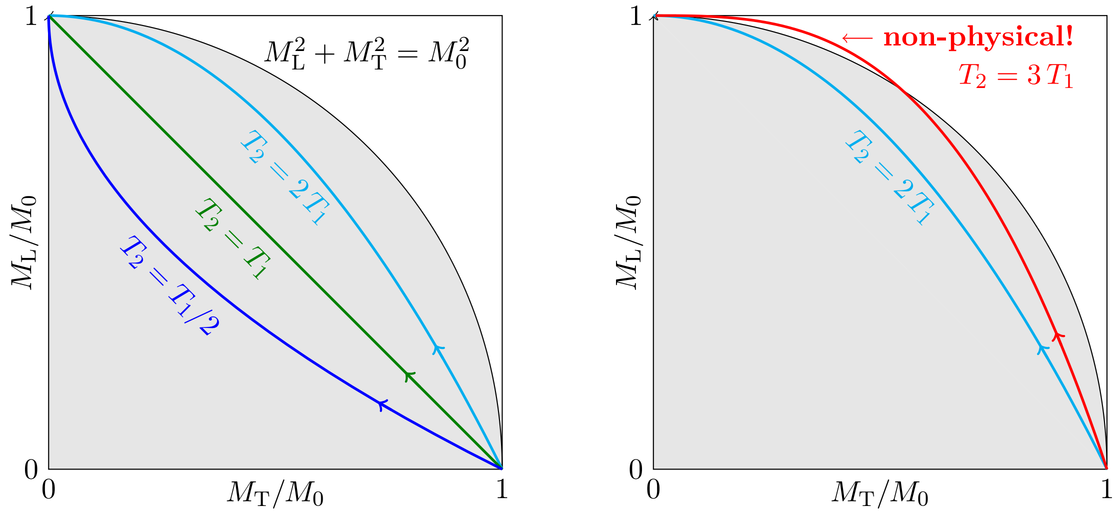

Figure 2: Evolution of the magnetization after a 90° rf pulse for $T_2 \leq 2\,T_1$ (left hand side) and $T_2 > 2\,T_1$ (red curve on right-hand side)

However, the right-hand side of this figure shows what's happening if T_2 becomes greater than $2\,T_1$ – in this case, $T_2 = 3\,T_1$ (red curve): Now the magnitude of the total magnetization vector increases above the physical limit of M_0, i. e., the red line crosses the black border of physically benign behavior!

In fact, it can be shown (see appendix if interested) that the maximum T_2 value, for which the red curve stays always below the black line, is exactly $T_2 = 2\,T_1$. And as so often, almost everything that is physically possible is also realized in nature (although the case $T_1 < T_2 < 2\,T_1$ is really extremely rare), as described by Malcolm H. Levitt in his highly recommendable NMR text book "Spin dynamics" (2nd ed., section 11.9.2, note 13):

> The case where $T_2 > T_1$ is encountered when the spin relaxation is caused by fluctuating microscopic fields that are predominantly transverse rather than longitudinal. One mechanism which gives rise to fields of this form involves

> the *antisymmetric component of the chemical shift tensor* (not to be confused with the CSA). [...] Molecular systems in which this mechanims is dominant are exceedingly rare (see F. A. L. Anet, D. J. O'Leary, C. G. Wade and R. D. Johnson, *Chem. Phys. Lett.*, **171**, 401 (1990)).

So, the answer to the title question is: No, T_2 can in fact be greater than T_1 in very special circumstances, but it can never be greater than $2\,T_1$.

Appendix

The maximum T_2 value, for which the red curve stays always below the black line, is exactly $T_2 = 2\,T_1$. This can be seen by analyzing the inequality

$$M_\mathrm{T}^2 + M_\mathrm{L}^2 = (M_0 \exp(-t/T_2))^2 + (M_0\,(1 - \exp(-t/T_1))^2 \leq M_0^2.$$

First, we divide by M_0^2 and set $T_2 = \alpha T_1$ as well as $\beta = \exp(-t/T_1)$, yielding

$$(\beta^{1/\alpha})^2 + (1-\beta)^2 = \beta^{2/\alpha} + 1 - 2\beta + \beta^2 \leq 1$$

which is (after subtraction of 1 and division by β)

$$\beta^{2/\alpha-1} - 2 + \beta \leq 0 \quad \text{or} \quad \beta^{2/\alpha-1} \leq 2 - \beta.$$

β is by definition (for positive t) between 0 and 1, so the right-hand side of the last inequality is a linear function descending from 2 to 1 (i. e. always ≤ 2). Its left-hand side has very different shapes depending on α: it is increasing from 0 to 1 for $0 < \alpha \leq 2$ (since then the exponent $2/\alpha - 1 \geq 0$); but it is going to infinity for $\beta \to 0$ if $\alpha > 2$ (since then the exponent $2/\alpha - 1 < 0$). So, the last inequality will not hold in the latter case for sufficiently small values of β, which means that non-physical behavior occurs if $\alpha > 2$ or, using the definition from above, if $T_2 > 2\,T_1$.

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