

# Is $T_2$ necessarily shorter than $T_1$ ?

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If you are familiar with MRI (or NMR in general), then probably also with the relaxation time constants  $T_1$  and  $T_2$ . These tissue-specific (or substance-specific) constants describe how fast the nuclear magnetization returns to its equilibrium value,  $M_0$ , after excitation by a pulsed radio-frequency (rf) field. Shortly summarized,  $T_1$  describes the exponential recovery of the longitudinal magnetization  $M_L$  (i. e., of the magnetization parallel to the external static magnetic field  $B_0$ ), and  $T_2$  describes the exponential decay of the transverse magnetization  $M_T$  (that is precessing in the plane orthogonal to  $B_0$ ).

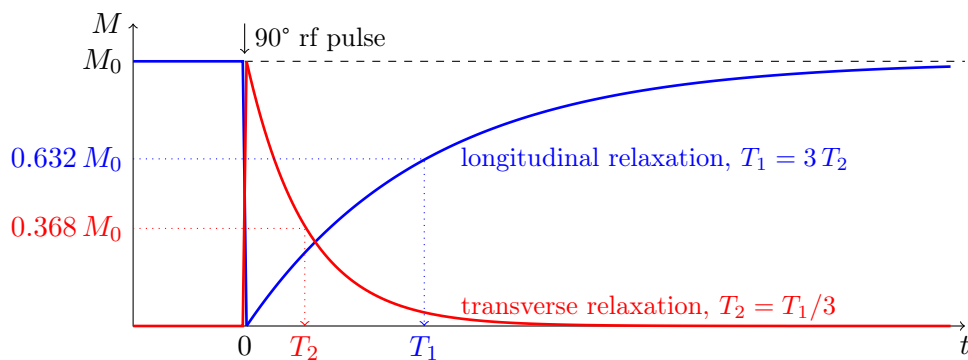


Figure 1: Longitudinal (blue) and transverse (red) relaxation of nuclear magnetization after a  $90^\circ$  rf pulse

Typically (as shown in the first figure),  $T_2$  values of tissue are considerably lower than  $T_1$  values, i. e., the transverse magnetization decays quicker than the longitudinal relaxation needs for recovery. For most tissues *in vivo*,  $T_1$  varies between about 300 ms and 3 s, while  $T_2$  varies between about 10 ms and 200 ms. Longer  $T_2$  relaxation times (up to about 3 s as well) are found for liquids.

So one may ask if there are good physical reasons for  $T_2$  values being shorter than or – at most – equal to  $T_1$ . As a physicist, I'd start with checking some extreme cases, e. g., assuming that  $T_2$  is *much* longer than  $T_1$ , i. e.  $T_2 \gg T_1$ . Then the longitudinal magnetization can fully recover while at the same time some transverse magnetization

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would be preserved. As a result, the magnitude of the total magnetization  $(M_L^2 + M_T^2)^{0.5}$  would become greater than the equilibrium value  $M_0$  – which is physically impossible.

To analyze these properties of  $T_1$  and  $T_2$  in more detail, longitudinal and transverse relaxation can also be plotted together in a diagram showing the transverse magnetization on the horizontal axis and the longitudinal magnetization on the vertical axis. These diagrams show the evolution of the magnetization (for experts: in a rotating frame of reference) after a  $90^\circ$  rf pulse (all trajectories start at the lower right corner of the diagram). The left-hand side of the following figure shows three cases  $T_2 = T_1/2$  (blue),  $T_2 = T_1$  (green), and  $T_2 = 2T_1$  [sic!] (cyan); and all three curves show a “benign” behavior in that they lie in the shaded area below the black circle segment  $M_L^2 + M_T^2 = M_0^2$ . This means that the magnitude of the total magnetization vector is always smaller than  $M_0$ .

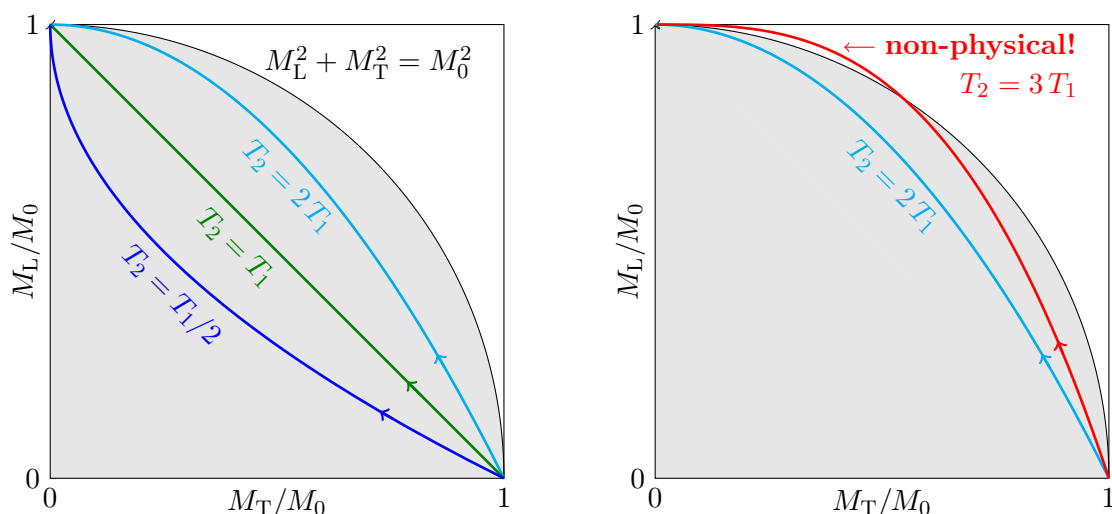


Figure 2: Evolution of the magnetization after a  $90^\circ$  rf pulse for  $T_2 \leq 2T_1$  (left hand side) and  $T_2 > 2T_1$  (red curve on right-hand side)

However, the right-hand side of this figure shows what’s happening if  $T_2$  becomes greater than  $2T_1$  – in this case,  $T_2 = 3T_1$  (red curve): Now the magnitude of the total magnetization vector increases above the physical limit of  $M_0$ , i. e., the red line crosses the black border of physically benign behavior!

In fact, it can be shown (see appendix if interested) that the maximum  $T_2$  value, for which the red curve stays always below the black line, is exactly  $T_2 = 2T_1$ . And as so often, almost everything that is physically possible is also realized in nature (although the case  $T_1 < T_2 < 2T_1$  is really extremely rare), as described by Malcolm H. Levitt in his highly recommendable NMR text book “Spin dynamics” (2nd ed., section 11.9.2, note 13):

The case where  $T_2 > T_1$  is encountered when the spin relaxation is caused by fluctuating microscopic fields that are predominantly transverse rather than longitudinal. One mechanism which gives rise to fields of this form involves

the *antisymmetric component of the chemical shift tensor* (not to be confused with the CSA). [...] Molecular systems in which this mechanism is dominant are exceedingly rare (see [F. A. L. Anet, D. J. O’Leary, C. G. Wade and R. D. Johnson](#), *Chem. Phys. Lett.*, **171**, 401 (1990)).

So, the answer to the title question is: No,  $T_2$  can in fact be greater than  $T_1$  in very special circumstances, but it can never be greater than  $2T_1$ .

## Appendix

The maximum  $T_2$  value, for which the red curve stays always below the black line, is exactly  $T_2 = 2T_1$ . This can be seen by analyzing the inequality

$$M_T^2 + M_L^2 = (M_0 \exp(-t/T_2))^2 + (M_0 (1 - \exp(-t/T_1)))^2 \leq M_0^2.$$

First, we divide by  $M_0^2$  and set  $T_2 = \alpha T_1$  as well as  $\beta = \exp(-t/T_1)$ , yielding

$$(\beta^{1/\alpha})^2 + (1 - \beta)^2 = \beta^{2/\alpha} + 1 - 2\beta + \beta^2 \leq 1$$

which is (after subtraction of 1 and division by  $\beta$ )

$$\beta^{2/\alpha-1} - 2 + \beta \leq 0 \quad \text{or} \quad \beta^{2/\alpha-1} \leq 2 - \beta.$$

$\beta$  is by definition (for positive  $t$ ) between 0 and 1, so the right-hand side of the last inequality is a linear function descending from 2 to 1 (i. e. always  $\leq 2$ ). Its left-hand side has very different shapes depending on  $\alpha$ : it is increasing from 0 to 1 for  $0 < \alpha \leq 2$  (since then the exponent  $2/\alpha - 1 \geq 0$ ); but it is going to infinity for  $\beta \rightarrow 0$  if  $\alpha > 2$  (since then the exponent  $2/\alpha - 1 < 0$ ). So, the last inequality will not hold in the latter case for sufficiently small values of  $\beta$ , which means that non-physical behavior occurs if  $\alpha > 2$  or, using the definition from above, if  $T_2 > 2T_1$ .

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