Double field strength gives double SNR – or not?

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Some time ago at lunch, we had a discussion about the advantages of high magnetic field strengths B_0 in MRI. We happily agreed that higher field strengths result in higher signal-to-noise ratios (SNR). But then several opinions surfaced about the exact quantitative relation between the SNR and B_0 – ranging from linear to quadratic and including some very specific exponents in between such as 7/4. It turns out that more than one correct answer exists ... and there are some surprising technical subtleties.

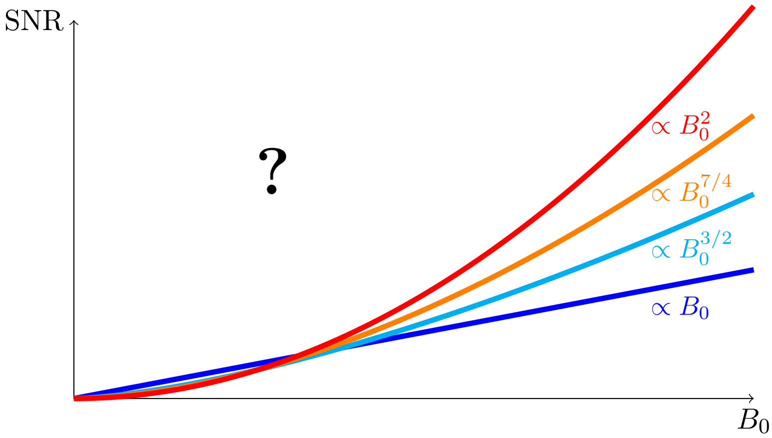

Figure 1: Discussed relationships of signal-to-noise ratio (SNR) and magnetic field strength B_0

As starting point of a quantitative discussion, we have to define what we consider as SNR in MRI: The SNR (sometimes given the symbol Ψ) is defined as the amplitude of the image signal (at some point or region of interest) divided by the standard deviation of the noise signal. (There are other SNR definitions used in other disciplines that involve squared amplitudes (signal power) and logarithms (resulting in decibels), but in MRI

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we generally use the very simple ratio of the signal amplitude and the noise standard deviation.)

To simplify things, we can divide the analysis of field strength and SNR into two parts: the discussion of the signal and of the noise. The signal part is rather straight forward: If the field strength B_0 increases, then

- the Larmor (precession) frequency $\omega = \gamma B_0$ of the nuclei increases proportionally with B_0 (the proportionality constant γ is the gyromagnetic ratio)
- and the nuclear magnetization M_N increases; for realistic field strengths and "normal" temperatures (around 300 K), the nuclear magnetization is approximately proportional to the field strength $M_\mathrm{N} \approx \chi_v B_0$, where χ_v is the nuclear magnetic (volume) susceptibility given by $\chi_v = \frac{(N/V)\gamma^2\hbar^2 I\,(I+1)}{3kT}$ (with N/V: spin density, I: nuclear spin quantum number, k: Boltzmann constant, T: sample temperature).

The measured signal S is the voltage induced in the radio-frequency (rf) receiver coil by the precessing nuclear magnetization. As known from the theory of electromagnetic induction, this induced voltage is proportional to both the (angular) frequency $\omega = \gamma B_0$ and the magnetization $M_\mathrm{N} = \chi B_0$, and taking both factors together, we find $S \propto B_0^2$.

So, if we stop at this point, we may hope for four times the SNR at double field strength. But we haven't considered the noise yet. And there the trouble begins ...

There are several sources of noise in MRI and depending on the setup, different sources can dominate the noise generation. Generally, the thermal noise voltage U_noise can be expressed as $U_\mathrm{noise} = \sqrt{4kT\,R\,\Delta f}$, where Δf is the signal bandwidth and R is a resistance associated to the rf receiver coil. For very small samples (milliliters), this resistance R is dominated by the actual coil resistance R_coil. R_coil is proportional to the square root of the Larmor frequency $R_\mathrm{coil} \propto \sqrt{\omega} \propto \sqrt{B_0}$ because of the rf *skin effect*, which reduces the penetration depth and, thus, the conducting area of the coil wires proportional to $1/\sqrt{\omega}$. Therefore, $U_\mathrm{noise} \propto R_\mathrm{coil}^{1/2} \propto \omega^{1/4} \propto B_0^{1/4}$ and the resulting SNR is proportional to $S/U_\mathrm{noise} \propto B_0^{7/4}$ in this case.

However, a different looking result is presented by A. Abragam in his classic monograph "The principles of nuclear magnetism" (1961). There (on p. 83) we find for the SNR $\Psi \propto \sqrt{Q}\,B_0^{3/2}$ with the coil quality factor Q. This is derived from the shunt resistance $R = QL\omega$ of a circuit (here the receive coil) with inductance L. This apparently contradicting result can be explained if the frequency dependence of the quality factor $Q \propto \sqrt{\omega}$ (for a Q-optimized solenoid coil) is considered, which then gives the same exponent of 7/4 as before.

Finally, for larger samples – such as human subjects in clinical MRI – additional noise is generated because of the inductive (or magnetic) losses associated with the electrical conductivity of the sample or tissue. Electrical power is dissipated in the sample proportional to the squared induced voltage in the sample, i. e., proportional to ω^2 and proportional to the squared current I^2 in the coil. This dissipation of power can therefore be expressed as an additional *apparent* coil resistance R_sample that is also proportional to the squared Larmor frequency $R_\mathrm{sample} \propto \omega^2$. Consequently, considering only this apparent coil resistance associated to the sample, we find $U_{\text{noise}} \propto R_{\text{sample}}^{1/2} \propto \omega \propto B_0$, and now the resulting SNR is proportional to $S/U_{\text{noise}} \propto B_0$.

These results (and detailed derivations) can be found in two classic MRI papers by D. I. Hoult and R. E. Richards (1976) and by D. I. Hoult and P. C. Lauterbur (1979).

An obvious question now is: Can't we simply measure the SNR dependence on the field strength to find out what's actually going on in MRI? Well, we can try, but there are several complications. One major problem is that we cannot use the same rf coils at different field strengths. There will generally be differences of the coil design for different field strengths and these must be expected to influence the measured SNRs. Another complication is the influence of the electromagnetic wavelengths in tissue that approach the dimensions of the samples at about 3 to 7 T. Furthermore, the relaxation times (T_1, T_2, T_2^*) change with B_0 and may influence SNR measurements.

Nevertheless, there are some publications reporting SNRs at different field strengths, e. g. by D. I. Hoult et al. (1986), J. T. Vaughan et al. (2001), C. Triantafyllou et al. (2005), and recently by R. Pohmann et al. (2016). They all show a clear increase of SNR with B_0, but there are still some discrepancies with respect to the exact dependency. Particularly the latest paper by R. Pohmann et al. demonstrates and discusses a slightly better than linear increase of SNR at high fields. But in conclusion and as rule of thumb, assuming an approximately linear relationship of SNR and B_0 appears still justified for large (clinical) MRI systems.

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