View-angle tilting MRI (part 3): Fewer approximations

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2020-09-29

This is a third (and presumably final) text about view-angle tilting (or VAT) MRI concluding the explanations posted on 2020-09-10 and on 2020-09-15. I don't really like the mathematical approximations that I used in my first text and, therefore, I would like to try again; the simple graphical visualization should correspond to an equally simple mathematical derivation . . .

Let's start as before with the MRI signal equation for an excited slice without any distortions

$$S(t) = \int \mathrm{d}x\, \rho(x, \hat{z}) \exp(-\mathrm{i}\gamma G_x x t) \tag{1}$$

with frequency-encoding direction x and frequency-encoding gradient G_x (phase encoding is generously omitted). The x integral is over the complete imaged object (but for a compact object that fits into our field of view, we can integrate from $-\infty$ to ∞). The slice-selection direction is denoted by $\hat{z}$, where I use the hatted ("^") variable to differentiate the undistorted (original) slice position $\hat{z}$ from the distorted position $z = z(x)$ analyzed below.

I will ignore the slice thickness, since the essence of the VAT technique can pretty well be explained without it. And everything gets really complicated if we consider varying slice thicknesses – or even slices split in several parts as illustrated in the following figure.

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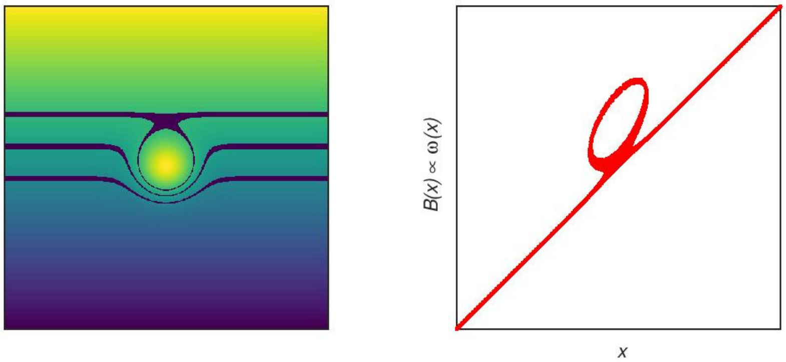

Figure 1: Slice selection in the presence of field inhomogeneities; left: excitation of three different slices with slice thickness varying due to inhomogeneity and with split geometry in the top slice; right: corresponding Larmor frequencies of the top slice during readout

It's useful to keep in mind the resonance relation between an excitation pulse with frequency ω_{rf} applied after switching on the slice-selection gradient $G_{z,\mathrm{slc}}$ and the slice position $\hat{z}$ (without field inhomogeneities):

$$\omega_{\mathrm{rf}} = \gamma G_{z,\mathrm{slc}} \hat{z} \quad \text{or} \quad \hat{z} = \frac{\omega_{\mathrm{rf}}}{\gamma G_{z,\mathrm{slc}}}. \tag{2}$$

If we now consider inhomogeneity-induced distortions of a slice (due to a field inhomogeneity, $\Delta B(x, z)$), then each original slice position $\hat{z}$ (corresponding to the excitation frequency ω_{rf}) is "moved" to a new position, $z(x)$, which is, in general, different for each x. The new position, $z(x)$, is the solution of the equation

$$\omega_{\mathrm{rf}} = \gamma \big(G_{z,\mathrm{slc}} z(x) + \Delta B(x, z(x)) \big) \tag{3}$$

for each (fixed) value of x. The relation to the original slice position, $\hat{z}$, can be expressed (after division by $\gamma G_{z,\mathrm{slc}}$) as

$$\frac{\omega_{\mathrm{rf}}}{\gamma G_{z,\mathrm{slc}}} = \hat{z} = z(x) + \frac{\Delta B(x, z(x))}{G_{z,\mathrm{slc}}}, \tag{4}$$

which can be re-arranged to

$$z(x) = \hat{z} - \frac{\Delta B(x, z(x))}{G_{z,\mathrm{slc}}}. \tag{5}$$

(Of course, the previous equation is not an explicit solution describing $z(x)$, since $z(x)$ still appears also on the right-hand side of the equation.)

If we go back to the MRI signal equation and include the distorted slice geometry, $z(x)$, the integral changes to

$$S(t) = \int \mathrm{d}x \, \rho(x, z(x)) \exp\Big(- \mathrm{i} \, \omega\big(x, z(x)\big) t \Big), \tag{6}$$

where, $\omega(x, z)$ is used to describe the Larmor frequencies during readout. Without field inhomogeneities, this is simply (and independent of z) $\omega(x, z) = \gamma G_x x$. The inhomogeneity, $\Delta B(x, z)$ changes the Larmor frequencies to $\omega\big(x, z(x)\big) = \gamma\left(G_x x + \Delta B(x, z(x))\right)$. If we also include the additional VAT readout gradient, $G_{z,\mathrm{VAT}}$, we get

$$\omega\big(x, z(x)\big) = \gamma\left(G_x x + \Delta B(x, z(x)) + G_{z,\mathrm{VAT}}\, z(x)\right). \tag{7}$$

We can now insert the expression for $z(x)$ derived above into the third addend of this last expression, which gives

$$\omega\big(x, z(x)\big) = \gamma\left(G_x x + G_{z,\mathrm{VAT}}\, \hat{z} + \left(1 - \frac{G_{z,\mathrm{VAT}}}{G_{z,\mathrm{slc}}}\right)\Delta B\big(x, z(x)\big)\right). \tag{8}$$

So, by setting $G_{z,\mathrm{VAT}} = G_{z,\mathrm{slc}}$ (as proposed for VAT MRI), the inner parenthesis vanishes and neither ΔB nor $z(x)$ (which implicitly contains ΔB as well) appears in the expression for the Larmor frequency. This means that the artifacts in readout direction (due to changed readout frequencies) are removed:

$$S(t) = \int \mathrm{d}x\, \rho\big(x, z(x)\big) \exp\left(-\mathrm{i}\gamma\left(G_x x + G_{z,\mathrm{VAT}}\, \hat{z}\right)\right). \tag{9}$$

Other artifacts caused by the changed slice geometry, $z(x)$, obviously remain (as indicated by the term $\rho(x, z(x))$ in the integral) and are *not* corrected by the VAT approach.

Since I've omitted the integration ($\int \mathrm{d}z \ldots$) over the finite slice thickness, the meaning of the slice coordinate $\hat{z}$ in the final results is not completely obvious. Actually, the appearance of $\hat{z}$ during readout corresponds to a tilted readout (that's why it's called *view-angle tilting*), which becomes clearer if one includes the omitted integration over the slice direction in the resulting MRI signal equation.

Keywords: MRI, artifacts, physics