No ADC Plateau in Restricted Diffusion:
ADC Strongly Time Dependent at Small $\tau$

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Introduction
In diffusion-weighted MRI (DWI), the random motion of molecules during a diffusion time $\tau$ causes a signal attenuation from which the apparent diffusion coefficient (ADC) can be calculated. For unrestricted diffusion, the mean displacement of molecules is proportional to the square root of $\tau$. If the diffusion is restricted, e.g. by cell membranes, the mean displacement is smaller and the calculated ADC will decrease with increasing diffusion time. Hence, the dependence between ADC and $\tau$ contains information about the confining geometry.

The ‘intuitive’ view published in many papers, e.g. [1-3], is that the restriction can be neglected if the diffusion time is sufficiently small (such that the expectation value of diffusion length is much smaller than the compartment size). We used computer simulations to study this dependence.

Methods
We used a one-dimensional computer model of diffusion with permeable membranes at equidistant positions. ADCs were calculated for diffusion times $\tau$ between 3 ms and 1 s using barrier reflection simulations [4]. We varied compartment sizes: 5, 10, 20, 30 $\mu$m and permeabilities: 0.1, 0.033, 0.01, 0.001 $\mu$m$^{-1}$. The resulting data were plotted using differently scaled diffusion time axes: against $\tau$, against the square root of $\tau$, and against the logarithm of $\tau$.

Results
The results of the simulations are shown in Figure 1. For diffusion times $\tau \rightarrow 0$, the ADC reaches its theoretical value corresponding to free diffusion (here $2 \times 10^{-3}$ mm$^2$/s). For long diffusion times $\tau \rightarrow \infty$, the ADC reaches a constant value depending on the permeability. However, the plots of ADC against $\tau$ show big differences at different scalings of the time axis, especially in the range of small $\tau$. Only the logarithmic plot has a turning point and a slope decreasing to zero for short diffusion times (the ‘plateau’). In the plot against the square root of $\tau$ the slope remains almost constant for short diffusion times and in the linear plot the slope increases strongly.

Discussion
The results show that there is no ADC plateau for short diffusion times when the data is displayed over the naturally scaled time axis, i.e. over the square root of the diffusion time. Even for very short diffusion times the restricted diffusion due to cell membranes causes a considerable reduction of the ADC. The logarithmic plot strongly emphasizes and extends the range of very short diffusion times that are not well accessible with normal DWI techniques.

References:

Figure 1: Results of diffusion simulations. The top row shows the calculated ADCs for different diffusion times from simulations with different permeabilities and a compartment size of 10 $\mu$m; the second row shows the calculated ADCs at different compartment sizes and a permeability of 0.033 $\mu$m$^{-1}$. The three columns contain the same data plotted against differently scaled time axes. Left column: logarithm of $\tau$, second column: square root of $\tau$, and right column: $\tau$.